Journal of Fluids and Structures 24 (2008) 1176–1184

JOURNAL OF FLUIDS AND STRUCTURES

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Regularization modeling for LES of separated boundary layer flow

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Received 5 January 2008; accepted 5 August 2008

Abstract

Regularization models for the turbulent stress tensor are applied to mixing and separated boundary layers. The Leray and the NS-α models in large-eddy simulation (LES) are compared to direct numerical simulation (DNS) and (dynamic) eddy-viscosity models. These regularization models are at least as accurate as the dynamic eddy-viscosity model, and can be derived from an underlying dynamic principle. This allows one to maintain central transport properties of the Navier-Stokes equations in the model and to extend systematically toward complex applications. The NS-α model accurately represents the small-scale variability, albeit at considerable resolution. The Leray model was found to be much more robust, allowing simulations at high Reynolds number. Leray simulations of a separated boundary layer are shown for the first time. The strongly localized transition to turbulence that arises under a blowing and suction region over a flat plate was captured accurately, quite comparable to the dynamic model. In contrast, results obtained with the Smagorinsky model, either with or without Van Driest damping, yield considerable errors, due to its excessive dissipation.

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Keywords: Turbulence; Separated flow; Large-eddy simulation; Regularization models; Eddy-viscosity

1. Introduction

In recent years considerable developments have been made in large-eddy simulation (LES) (Meneveau and Katz, 2000; Sagaut, 2001; Geurts, 2003). New, mathematically rigorous, sub-filter models have been constructed (Foias et al., 2001; Geurts and Holm, 2003). These hold promise to achieve robust LES that can reliably treat flows of challenging complexity. In this paper, we present the application of regularization modeling to separated boundary layer flow.

Separated boundary layer flow arises in various technological applications. Often, separation occurs as a result of sharp corners in the flow domain, e.g., flow over a step or a cavity (Friedrich and Arnal, 1990). However, already the

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doi:10.1016/j.jfluidstructs.2008.08.006

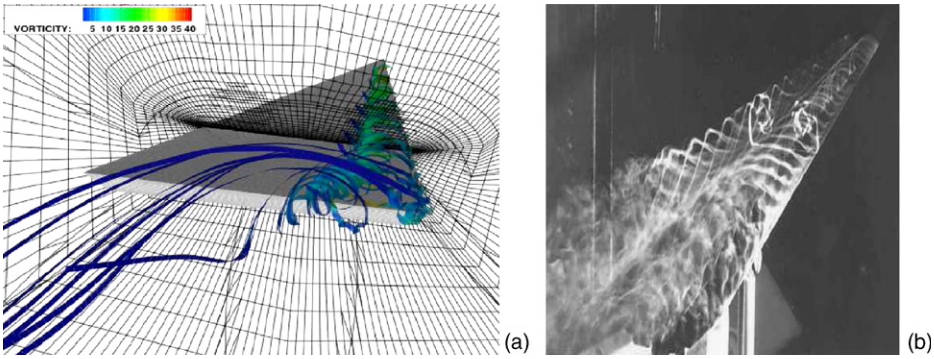

Fig. 1. Illustration of a separated wing-tip vortex in flow over a delta-wing. In (a) results of a large-eddy simulation at $\mathrm{Re}_c = 10^5$ are shown (van der Bos and Geurts, 2008), which compare qualitatively with (b) experimental flow-visualization of flow at $\mathrm{Re}_c = 2 \times 10^5$ (Riley and Lowson, 1998).

presence of a suitable pressure distribution at some distance from a smooth solid wall may trigger separation (Alam and Sandham, 2000). In such cases the flow-physics of an attached boundary layer occurs side-by-side with that of a free shear layer. The instability mechanisms in these 'constituent flows' differ widely, resulting in a variety of length- and time-scales that need to be captured simultaneously. In this paper, we will address this multiscale problem with LES.

Frequently, separation of flow from a solid object is a precursor of transition to turbulence. An example is shown in Fig. 1 displaying wing-tip vortices over a delta-wing. This illustrates flow separation as well as coherent trailing vortices. The latter form a challenging problem in air-traffic control, as these flow structures dictate safe separation distances between different aircrafts (Liu, 1992). Other examples include vortex shedding from slender bodies (Bearman, 1984), inducing potentially strong flow structure interactions, such as occur in offshore risers. An understanding of the basic instability mechanisms, their nonlinear growth and their range of dynamic scales are essential to achieve a robust and reliable control of such flows. Large-scale computation can offer important support for designing new flow-control strategies. Here, we will focus on a separated flow, induced by blowing and suction and ask the question to what extent this flow can be treated with regularization modeling of the turbulent stresses.

In Section 2 we introduce LES and regularization modeling for the turbulent stress tensor. In Section 3 regularization modeling is applied to a turbulent shear layer. Subsequently, Leray regularization is applied to separated boundary layer flow and compared to direct numerical simulation (DNS) and eddy-viscosity models in Section 4. Concluding remarks are in Section 5.

2. Regularization modeling for LES

In this section we review the filtering approach to LES and discuss the regularization and eddy-viscosity modeling of the turbulent stresses.

Filtering the Navier–Stokes equations requires a low-pass spatial filter L. Often, a convolution filter is adopted, which, in one spatial dimension, associates the filtered velocity $\overline{u}$ with the unfiltered velocity u through

$$\overline{u} = L(u) = \int_{-\infty}^{\infty} G(x-\xi, \Delta) u(\xi)\, \mathrm{d}\xi, \tag{1}$$

with normalized filter-kernel $G(z, \Delta)$. This filter-kernel is characterized by an externally specified filter-width Δ (van der Bos and Geurts, 2005).

For incompressible fluids, the application of the filter L to the continuity and Navier–Stokes equations leads to

$$\partial_j \overline{u}_j = 0 \quad \text{and} \quad \partial_t \overline{u}_i + \partial_j(\overline{u}_j \overline{u}_i) + \partial_i \overline{p} - \frac{1}{\mathrm{Re}} \partial_{jj} \overline{u}_i = -\partial_j(\overline{u_i u_j} - \overline{u}_j \overline{u}_i). \tag{2}$$

Here ∂_t (resp. ∂_j) denotes partial differentiation with respect to time t (resp. spatial coordinate x_j). Summation over repeated indices is implied. The component of the filtered velocity in the x_j direction is $\overline{u}_j$ and $\overline{p}$ is the filtered pressure. Finally, Re denotes the Reynolds number of the flow. In (2) the Navier–Stokes operator is applied to the filtered solution $\{\overline{u}_j, \overline{p}\}$ on the left-hand side. On the right-hand side, the divergence of the turbulent stress tensor $\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j$ arises. This tensor cannot be calculated from the filtered solution alone and constitutes a closure problem.

One of the issues in LES is the capturing of the primary dynamical effects of τ_{ij} in terms of so-called 'sub-filter' model tensors m_{ij}.

LES sub-filter models m_{ij} are often proposed either on the basis of their dissipative nature, or in view of the scale-similarity property of τ_{ij} in an inertial range (Meneveau and Katz, 2000). As further guidance in the construction of models, one may incorporate mathematical properties of the filtered equations such as realizability (Vreman et al., 1994), algebraic identities (Germano et al., 1991) or approximate inversion of the filter (Geurts, 1997; Stolz and Adams, 1999). While realizability conditions impose bounds on model parameters, algebraic properties such as Germano's identity, possibly combined with filter-inversion (Kuerten et al., 1999), lead to the class of dynamic sub-filter models.

2.1. Eddy-viscosity models

Much of turbulence phenomenology is captured in the Kolmogorov picture, in which kinetic energy cascades in an average sense through an inertial range toward ever smaller scales (Kolmogorov, 1991; Frisch, 1995). When a sufficiently wide filter is applied to a turbulent solution, (virtually) all *molecular* dissipation associated with the smallest scales is removed. In LES this is compensated by the introduction of an 'extra' eddy-viscosity contribution. Dissipation of turbulent kinetic energy in this way was first parameterized by using the Smagorinsky (1963) model:

$$\tau_{ij} \to m_{ij}^S = -(C_S \Delta)^2 |S| S_{ij}, \tag{3}$$

where C_S denotes Smagorinsky's constant, $S_{ij} = \partial_i \overline{u}_j + \partial_j \overline{u}_i$ is the rate of strain tensor and $|S|^2 = S_{ij} S_{ij}/2$ is its squared magnitude.

The Smagorinsky model is known to display low levels of correlation with τ_{ij} (Geurts, 2003) and often leads to excessive dissipation, especially near solid walls (Piomelli et al., 1990). This may even hinder a complete transition to turbulence (Vreman et al., 1997). The central problem that arises is how the eddy-viscosity coefficient should be specified in accordance with the evolving flow such that this problem is circumvented. A well-known and elegant way to approach this without unduly introducing *ad hoc* parameters is based on Germano's identity (Germano et al., 1991) which provides the basis for the dynamic sub-filter modeling procedure. Here, we will use a standard implementation of the dynamic procedure to arrive at an eddy-viscosity model with proper damping of dissipation near solid walls and sharp gradients in the mean field (Meyers et al., 2005). For further details we refer to (Geurts, 2003).

Dynamic sub-filter models have contributed in many ways to the understanding of turbulent flow in complex situations. However, dynamic models are hampered in several ways. First, the dynamic procedure is quite expensive in view of the explicit filter operations that need to be included. Moreover, the implementation contains various *ad hoc* elements or inaccurate assumptions such as the independence of the dynamic coefficient on the filter-level, or the well-known 'clipping' of negative eddy-viscosity, required to ensure stability of a simulation. Finally, an extension to flows involving complex physics and/or developing in complex flow-domains is difficult since no systematic framework exists for this purpose. For these reasons, we turn to the recently proposed alternative of regularization modeling next (Geurts and Holm, 2003).

2.2. Regularization strategy to sub-filter closure

We describe two regularization principles and derive the associated sub-filter models in case the LES filter L has a formal inverse L^{-1} (Geurts and Holm, 2003). We consider Leray (1934) regularization and the Navier–Stokes-α (NS-α) approach (Foias et al., 2001).

Historically, the first regularized flow description is due to Leray (1934). Although this regularization was introduced for entirely different reasons, we may reinterpret the Leray proposal in terms of its implied sub-filter-model. In Leray regularization, one alters the convective fluxes into $\overline{u}_j \partial_j u_i$, i.e., the solution $\mathbf{u}$ is convected with a smoothed velocity $\overline{\mathbf{u}}$. Consequently, the nonlinear effects are reduced by an amount governed by the smoothing properties of the filter operation, L. The governing Leray equations are

$$\partial_j \overline{u}_j = 0; \quad \partial_t u_i + \overline{u}_j \partial_j u_i + \partial_i p - \frac{1}{\mathrm{Re}} \Delta u_i = 0. \tag{4}$$

We may eliminate $\mathbf{u}$ by assuming $\overline{\mathbf{u}} = L(\mathbf{u})$ and $\mathbf{u} = L^{-1}(\overline{\mathbf{u}})$. Consequently, one may readily obtain

$$\partial_t \overline{u}_i + \partial_j (\overline{u}_j \overline{u}_i) + \partial_i \overline{p} - \frac{1}{\mathrm{Re}} \Delta \overline{u}_i = -\partial_j (m_{ij}^L), \tag{5}$$

with an implied asymmetric Leray model,

$$m_{ij}^L = L(\overline{u}_j L^{-1}(\overline{u}_i)) - \overline{u}_j\overline{u}_i = \overline{\overline{u}_j u_i} - \overline{u}_j\overline{u}_i. \tag{6}$$

The reconstructed solution u_i is found from an approximate inversion L^{-1}. For this purpose one may use a number of methods, e.g., polynomial inversion (Geurts, 1997), geometric series expansions (Stolz and Adams, 1999) or exact numerical inversion of Simpson top-hat filtering (Kuerten et al., 1999), as adopted here.

An extended regularization principle that is based on a filtered Kelvin circulation theorem is known as the NS-α approach (Foias et al., 2001; Holm et al., 1998). From the filtered Kelvin principle, we may obtain the Euler–Poincaré equations governing the smoothed fluid dynamics (Holm et al., 1998):

$$\partial_t u_j + \overline{u}_k \partial_k u_j + u_k \partial_j \overline{u}_k + \partial_j p - \partial_j\left(\frac{1}{2}\overline{u}_k u_k\right) - \frac{1}{\mathrm{Re}}\Delta u_j = 0. \tag{7}$$

Comparison with the Leray regularization principle in (4) reveals two additional terms in (7). These terms guarantee the regularized flow to be consistent with the mollified Kelvin circulation theorem. The Euler–Poincaré equations (7) can be rewritten in the form of the LES template, giving rise to the implied sub-filter model:

$$\partial_t \overline{u}_i + \partial_j(\overline{u}_j\overline{u}_i) + \partial_i \overline{p} - \frac{1}{\mathrm{Re}}\Delta\overline{u}_i = -\partial_j(\overline{\overline{u}_j u_i} - \overline{u}_j\overline{u}_i) - \frac{1}{2}(\overline{u_j\partial_i\overline{u}_j - \overline{u}_j\partial_i u_j}). \tag{8}$$

3. Regularized turbulent mixing

In this section we present LES of a turbulent mixing layer, validating the regularization modeling of the turbulent stress tensor for flow with a strongly localized gradient in the mean flow.

We adopt a fourth order accurate skew-symmetric finite volume method in combination with explicit Runge–Kutta time-stepping (Geurts, 2003; Vreman et al., 1997; Meyers et al., 2003). The three-dimensional temporal mixing layer is solved at a Reynolds number based on upper stream velocity and half the initial vorticity thickness of 50. The governing equations are solved in a cubic geometry of side $\ell = 59$. Periodic boundary conditions are imposed in the streamwise (x_1) and spanwise (x_3) direction, while in the normal (x_2) direction the boundaries are free-slip walls. Initially, we assume a mean flow with constant pressure p, $u_1 = \tanh(x_2)$ and $u_2 = u_3 = 0$. To initiate turbulence, two-and three-dimensional instability modes were superimposed. Further details may be found in Geurts and Holm (2003); Vreman et al. (1997) and Geurts and Holm (2006).

A qualitative test of sub-filter models is obtained by comparing the prediction of dominant flow-structures in instantaneous solutions. As illustrated, the filtered DNS prediction of the vertical velocity and the corresponding Leray and NS-α results are shown in the turbulent regime in Fig. 2. Both the Leray and NS-α models capture the 'character' of the filtered solution quite well. While the Leray solution appears to slightly under-predict the small scales, the NS-α solution corresponds very closely to the filtered DNS findings, retaining much of the small scale variability. These instantaneous predictions are both much better than those obtained with (dynamic) eddy-viscosity models which turn out to be too smooth (Vreman et al., 1997).

To assess the quality of the Leray and NS-α models we turn to the prediction of mean and fluctuating properties next. The evolution of the momentum thickness δ is shown in Fig. 3(a) at $\Delta/h = 6$ (Geurts and Fröhlich, 2002). The prediction of δ is most accurate with the NS-α model, followed by the Leray model and then by the dynamic model. To test the prediction of the smaller scales we consider velocity fluctuations $v_i = \overline{u}_i - \langle\overline{u}_i\rangle$, where $\langle\cdot\rangle$ denotes averaging over the homogeneous directions. As seen in Fig. 3(b) both regularization models provide a fair approximation of the streamwise turbulent intensity, compared to filtered DNS, considerably more accurate than the dynamic model. To put this further into perspective, an earlier study (Vreman et al., 1997) established the dynamic model to be among the more accurate models compared to the similarity model and dynamic mixed models.

These simulation results clearly illustrate the improvements in the description of the flow-physics by the NS-α model in case the spatial resolution is adequate. A grid-convergence study showed that regularization models are more sensitive to mesh-coarsening than the dynamic model. The Leray model appears to give a good compromise between robustness and accuracy, also in the limit of very high Reynolds numbers (Geurts and Holm, 2006). In the sequel we concentrate on the Leray model and apply it in the context of separated boundary layer flow.

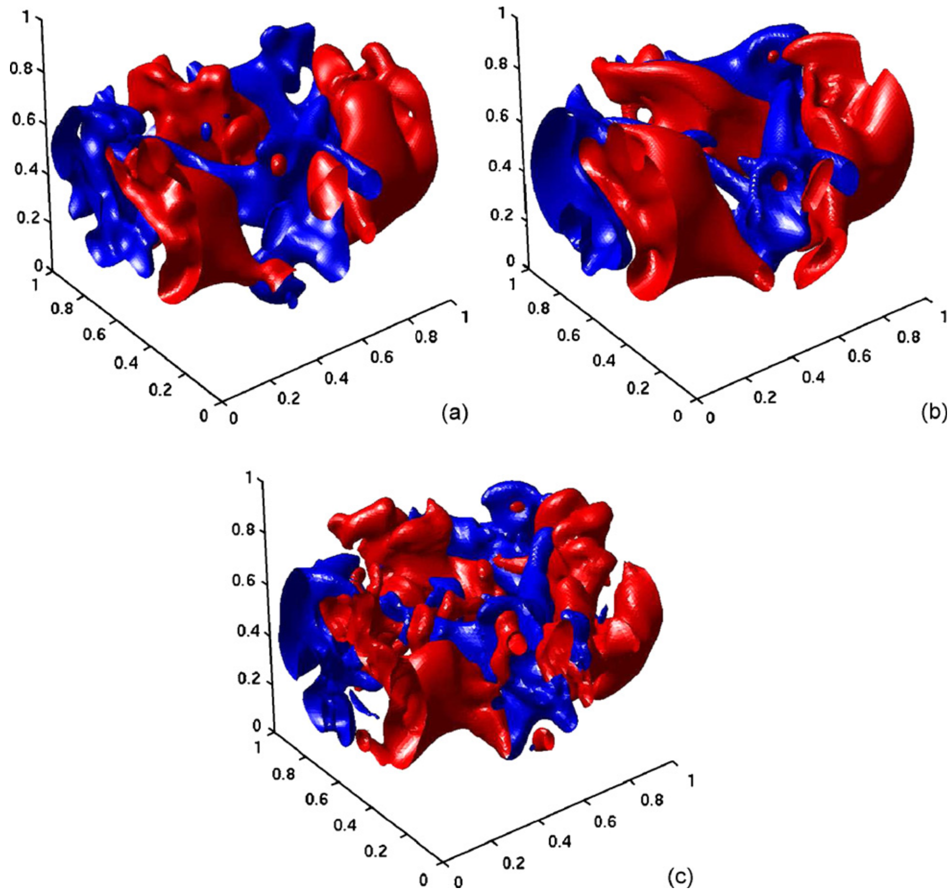

Fig. 2. Vertical velocity at $t = 80$; red (blue) contours indicate $u_2 = 0.3$ ($u_2 = -0.3$). Filtered DNS at 192^3: (a) Leray; (b) NS-α; (c) using $\Delta = \ell/16$ and a 96^3 grid.

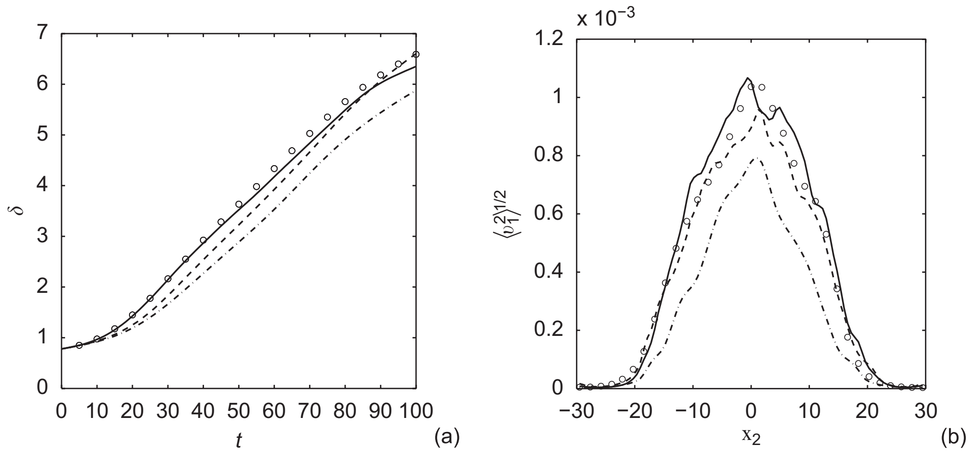

Fig. 3. Momentum thickness δ (a) and streamwise turbulent intensity $\langle v_1^2 \rangle^{1/2}$ at $t = 80$, using Re $= 50$ and $\Delta = \ell/16$ for NS-α (solid), Leray (dash) and dynamic model (dash–dot). LES at 96^3 are compared with filtered DNS (open circles).

4. Regularized separated turbulent boundary layer

In this section we present the simulation of a turbulent separated boundary layer, comparing the Leray regularization to eddy-viscosity models and DNS.

The separated boundary layer is considered in a computational domain as sketched in Fig. 4. The treatment of the boundaries requires special attention. We closely follow Poinsot and Lele (1992), with some specific extensions (Wasistho et al., 1997a). At the inflow, a Blasius profile is introduced (Schlichting, 1979), to which instability modes are superimposed. At the outflow characteristic non-reflecting boundary conditions are used in combination with a buffer zone in which turbulent fluctuations are directly damped (Wasistho et al., 1997a). This approach was validated as described in Wasistho et al. (1997b). In the spanwise direction periodic conditions are applied. Finally, no-slip conditions are used at the solid wall and suction/blowing conditions in the far-field, to stimulate a localized transition associated with the separation bubble that develops (Alam and Sandham, 2000).

In Fig. 5 an impression of the vorticity field $\boldsymbol{\omega} = \nabla \times \mathbf{u}$ is given, as obtained with DNS at a resolution of $512 \times 128 \times 128$ in the streamwise, spanwise and normal direction. The dimensions of the flow-domain are $350 \times 20 \times 20$ and the Reynolds number equals 330, based on the inflow displacement thickness (Wasistho et al., 1997a). The development

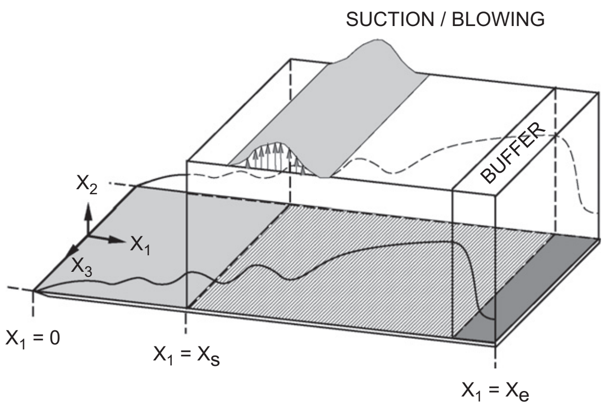

Fig. 4. Computational domain including a blowing and suction agitation and downstream outflow boundaries in combination with a buffer region (Wasistho et al., 1997a).

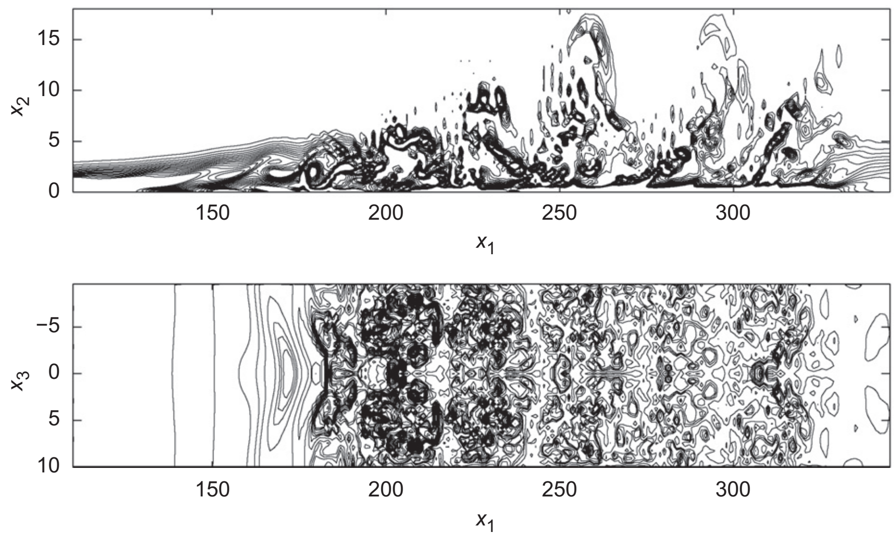

Fig. 5. Direct numerical simulation of instantaneous spanwise vorticity contours in the plane $x_3 = 0$ (upper figure) and a top view of the total vorticity (lower figure).

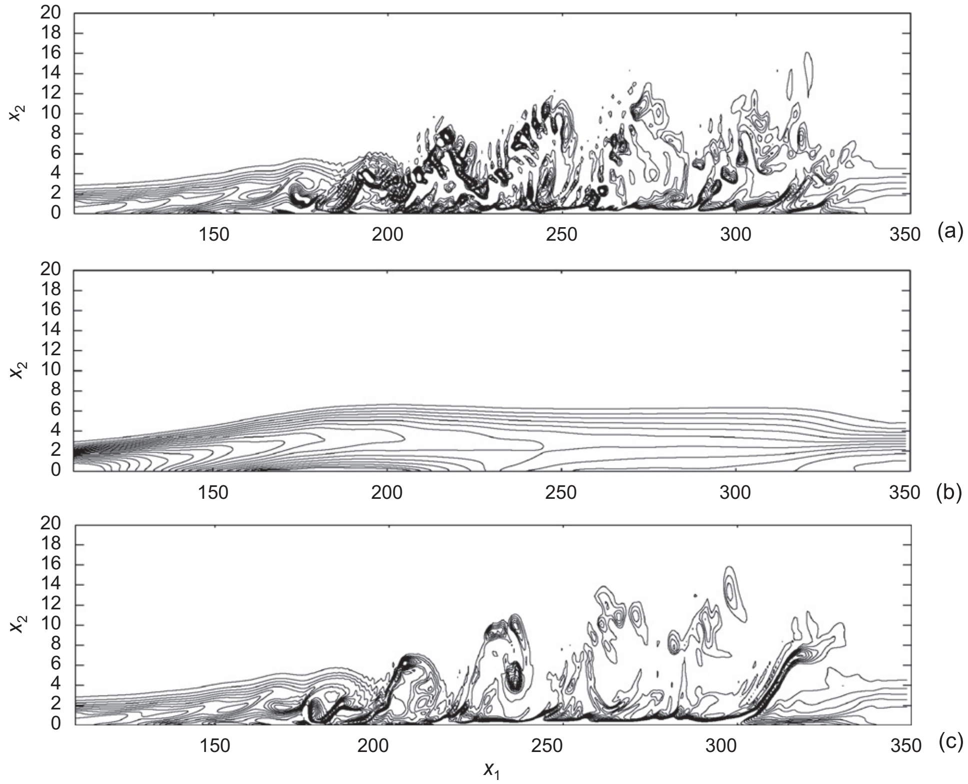

Fig. 6. Instantaneous spanwise vorticity obtained (a) with DNS, (b) LES based on Smagorinsky's model with Van Driest damping and (c) Leray model.

of a separated high shear layer is observed, with roll up structures seen in the temporal mixing layer. The separation of the shear layer results in a strong transition and shedding of vortices from the separation bubble. Downstream, a gradual recovery of a zero-pressure gradient boundary layer arises. Finally, the flow relaminarizes inside the outflow buffer-domain.

In Fig. 6 we compare vorticity fields as obtained with DNS, and LES based on the Smagorinsky (1963) model and Leray regularization at a resolution of $128 \times 64 \times 64$. In an attempt to compensate for the too high levels of eddy-viscosity near the solid wall (Piomelli et al., 1990) we included Van Driest (1956) damping in the wall-normal direction. However, even with Van Driest damping, the transition to turbulence is blocked by the Smagorinsky model. The strong gradients that also exist in the streamwise direction, particularly near the separation bubble are not damped adequately, resulting in large errors in the simulation. The situation is much improved with the Leray model, which captures the primary flow features quite well. We also investigated the dynamic eddy-viscosity model and observed nearly identical results as obtained with the Leray model. In the dynamic model the spanwise averaged dynamic coefficient was found to be near zero upstream of the separation bubble, and approached values slightly above 0.01 in the turbulent region downstream of transition. The dynamic coefficient also decreases to zero when approaching the solid wall. This reflects the self-contained damping of the eddy-viscosity in the dynamic procedure.

To quantify the accuracy with which the boundary layer flow is represented we show the skin friction s and the shape factor H in Fig. 7. The Smagorinsky solutions, with and without Van Driest damping, display significant errors, underlining the exaggerated dissipation of this model. The dynamic model and the Leray model generate accurate predictions for these mean flow properties, at considerably reduced computational costs, compared to DNS.

5. Concluding remarks

We reviewed the filtering approach to LES and applied Leray and NS-α regularization to close the turbulent stresses. These models are derived from an assumed dynamic principle that is aimed to coarsen the flow description while

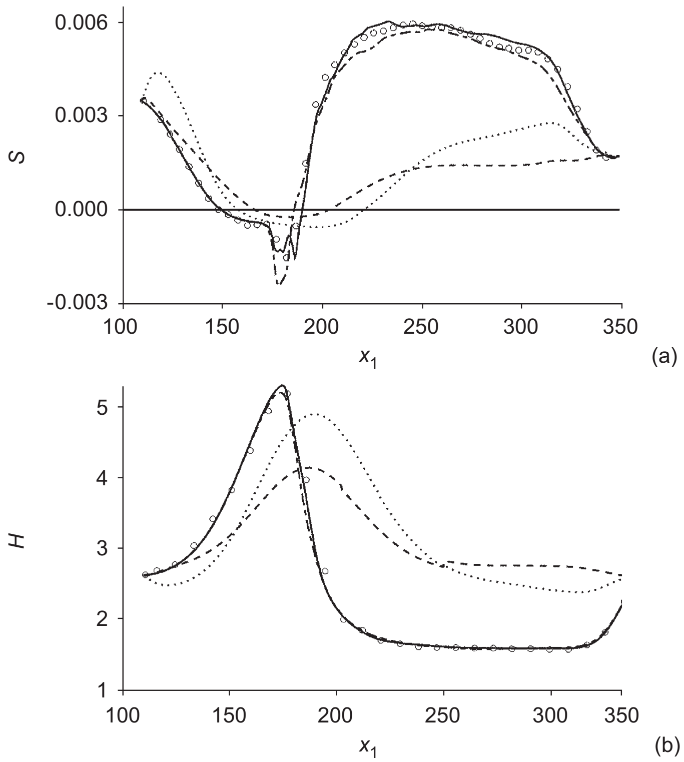

Fig. 7. (a) Skin friction s and (b) shape factor H as predicted by DNS (solid), dynamic eddy-viscosity model (dash–dot), Smagorinsky model without (dot) and with (dash) Van Driest damping, and the Leray model (○).

maintaining a number of important transport properties of the smoothed equations (Holm et al., 1998). To date, these models were applied to the simplest canonical flow-problems, i.e., homogeneous, isotropic turbulence (Chen et al., 1999; Geurts et al., 2008), and flow in a temporally developing mixing layer (Geurts and Holm, 2006). We reviewed regularized mixing and showed that the results are comparable to and often better than those obtained with the dynamic eddy-viscosity model. The NS-α model was found most accurate but requires considerable resolution. The Leray model was slightly less accurate but much more robust in turbulent mixing.

In this paper, the Leray model was applied for the first time to a separated boundary layer. We observed that the strongly localized transition to turbulence that arises under the blowing and suction region was captured accurately, quite comparable to the dynamic model. In contrast, results obtained with the Smagorinsky model, either with or without Van Driest damping were found to yield considerable errors, due to excessive dissipation.

The application of the Leray model to flows of high complexity is subject of ongoing research. A particular challenge is the resolution of the near-wall behavior at strongly increased Reynolds numbers. The dynamic Leray principle induces its own near-wall treatment that requires high resolution. The Leray model was found to yield too many small scales near the solid wall in the case where the Reynolds number is increased considerably. Modifications of the Leray principle appear to be needed to achieve high accuracy as well as acceptable computational costs for high Reynolds boundary layers. This is currently being studied.

References

Alam, M., Sandham, N.D., 2000. Direct numerical simulation of 'short' laminar separation bubbles with turbulent reattachment. Journal of Fluid Mechanics 403, 223–246.

Bearman, P.W., 1984. Vortex shedding from oscillating bluff bodies. Annual Review of Fluid Mechanics 16, 195–222.

van der Bos, F., Geurts, B.J., 2005. Commutator errors in the filtering approach to large eddy simulation. Physics of Fluids 17, 035108.

van der Bos, F., Geurts, B.J., 2008. Computational error-analysis of discontinuous Galerkin discretization applied to large-eddy simulation of homogeneous turbulence. Computers and Fluids, submitted for publication.

Chen, S., Foias, C., Holm, D.D., Olson, E., Titi, E.S., Wynne, S., 1999. A connection between the Camassa–Holm equations and turbulent flows in channels and pipes. Physics of Fluids 11, 2343–2353.

Foias, C., Holm, D.D., Titi, E.S., 2001. The Navier–Stokes-alpha model of fluid turbulence. Physica D 152, 505–512.

Friedrich, R., Arnal, M., 1990. Analyzing turbulent backward-facing step flow with the low-pass-filtered Navier–Stokes equations. Journal of Wind Engineering and Industrial Aerodynamics 35, 101–117.

Frisch, U., 1995. Turbulence: The Legacy of A.N. Kolmogorov. Cambridge University Press, Cambridge.

Germano, M., Piomelli, U., Moin, P., Cabot, W.H., 1991. A dynamic sub-filter-scale eddy viscosity model. Physics of Fluids 3, 1760–1764.

Geurts, B.J., 1997. Inverse modeling for large eddy simulation. Physics of Fluids 9, 3585–3589.

Geurts, B.J., 2003. Elements of Direct and Large-Eddy Simulation. R.T. Edwards, Flourtown.

Geurts, B.J., Fröhlich, J., 2002. A framework for predicting accuracy limitations in large eddy simulation. Physics of Fluids 14, L41.

Geurts, B.J., Holm, D.D., 2003. Regularization modeling for large eddy simulation. Physics of Fluids 15, L13.

Geurts, B.J., Holm, D.D., 2006. Leray and NS-α modeling of turbulent mixing. Journal of Turbulence 7 (10), 1–33.

Geurts, B.J., Kuczaj, A.K., Titi, E.S., 2008. Regularization modeling for large-eddy simulation of homogeneous isotropic decaying turbulence. Journal of Physics A: Math. Theor. 41, 344008.

Holm, D.D., Marsden, J.E., Ratiu, T.S., 1998. The Euler–Poincaré equations and semidirect products with applications to continuum theories. Advances in Mathematics 137, 1–67.

Kolmogorov, A.N., 1991. The local structure of turbulence in an incompressible fluid with very large Reynolds numbers. Proceeding Royal Society London, 434, 1890-1903 (first published: 1941, Doklady Akademia Nauk SSSR 30, 301–314).

Kuerten, J.G.M., Geurts, B.J., Vreman, A.W., Germano, M., 1999. Dynamic inverse modeling and its testing in large-eddy simulations of the mixing layer. Physics of Fluids 11, 3778–3786.

Leray, J., 1934. Sur les movements d'un fluide visqueux remplissant l'espace. Acta Mathematica 63, 193–246.

Liu, H.-T., 1992. Effects of ambient turbulence on the decay of a trailing vortex wake. Journal of Aircraft 29, 255–263.

Meneveau, C., Katz, J., 2000. Scale-invariance and turbulence models for large eddy simulation. Annual Review of Fluid Mechanics 32, 1–32.

Meyers, J., Geurts, B.J., Baelmans, M., 2003. Database analysis of errors in large eddy simulation. Physics of Fluids 15, 2740–2754.

Meyers, J., Geurts, B.J., Baelmans, M., 2005. Optimality of the dynamic procedure for large-eddy simulation. Physics of Fluids 17, 045108.

Piomelli, U., Zang, T.A., Speziale, C.G., Hussaini, M.Y., 1990. On the large-eddy simulation of transitional wall-bounded flows. Physics of Fluids A 2, 257–273.

Poinsot, T.J., Lele, S.K., 1992. Boundary conditions for direct simulations of compressible viscous flows. Journal of Computational Physics 101, 104–128.

Riley, A.J., Lowson, M.V., 1998. Development of a three dimensional free shear layer. Journal of Fluid Mechanics 369, 49–89.

Sagaut, P., 2001. Large-Eddy Simulation for Incompressible Flows; An Introduction. Scientific Computation. Springer, Berlin.

Schlichting, H., 1979. Boundary Layer Theory, seventh ed. Mc Graw-Hill, New York.

Smagorinsky, J., 1963. General circulation experiments with the primitive equations. Monthly Weather Review 91, 99–109.

Stolz, S., Adams, N.A., 1999. An approximate deconvolution procedure for large-eddy simulation. Physics of Fluids 11, 1699–1703.

Van Driest, E.R., 1956. On turbulent flow near a wall. Journal of Aerospace Science 23, 1007–1021.

Vreman, A.W., Geurts, B.J., Kuerten, J.G.M., 1994. Realizability conditions for the turbulent stress tensor in large eddy simulation. Journal of Fluid Mechanics 278, 351–369.

Vreman, A.W., Geurts, B.J., Kuerten, J.G.M., 1997. Large-eddy simulation of the turbulent mixing layer. Journal of Fluid Mechanics 339, 357–381.

Wasistho, B., Geurts, B.J., Kuerten, J.G.M., 1997a. Accurate boundary conditions for time-dependent flow over a flat plate. Computers and Fluids 26, 713–728.

Wasistho, B., Geurts, B.J., Kuerten, J.G.M., 1997b. Numerical simulation of separated boundary layer flow. Journal of Engineering Mathematics 32, 179–193.